Stable boundary of virtual mass for a haptic admittance display system

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Abstract. This paper discusses the stable range of a 1 degree-of-freedom haptic system in admittance display. The human operator is regarded as a combination of passive impedance and active input in the system. And the virtual environment is assumed to be passive. Through analogous treatment between the haptic system and electrical network system, port theory was used to analyze the system. Considering a simple but typical virtual environment of a pure mass, the stable boundary of the virtual mass was derived. Matlab simulation proved the validity of the proposed method. The method can be applied for performance evaluation in the device design.

1 Introduction

Stability and transparence are important specifications in a haptic system, but they counteract each other. Analysis of factors affecting system performance can guide the trade-off between those contradicting requirements. Basically haptic implementation falls into two modes: impedance display which measure motion and display force [1,3], and admittance display which measure force and display motion [4-6]. Assuming the virtual environment to be a parallel combination of spring and damping, Colgate examined the stability problem of a 1-DOF haptic impedance display system [7]. There is little work on the stable condition of admittance display when simulating a virtual mass. Considering that mass manipulation is a common kind of environment manipulation, this paper focuses on the stability analysis of a 1-DOF haptic admittance display system when simulating mass manipulation.

The model of the human operator plays an important role in the analysis of haptic system. Some researchers treated the human operator as unknown passive impedance [7-8]. Other researchers treated the human operator as a linear combination of mass, spring and damping [3,9]. This paper will regard the human operator as a combination of passive impedance and active input. The active part acts as the system motion input, and the passive part acts as the controller.

In the following section the system model is built. Stable criteria and stable boundary of virtual mass are examined in section 3. Then section 4 tests the method and the analytical results through simulation. Section 5 concludes the results.

2 System model based on mechanical and electrical analogy

Figure 1 shows the basic components of a haptic system: the human operator, the haptic device and the virtual environment. There is direct force and motion interaction between the operator and the haptic device, and information interaction between the haptic device and the virtual environment.

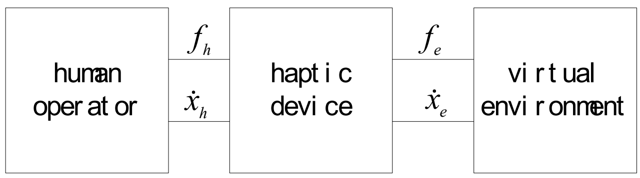

Fig. 1. The structure of haptic system

We consider a haptic device shown in Figure 2 ([10]). The configuration of the mechanism is selected to accommodate the shape of one kind of surgical instrument, which is used in a robot-assisted fiberscope surgery system developed in our lab. This haptic system can be used as the human-machine interface to control the slave robot of the system. During haptic simulation, operator pushes or pulls the slider via two finger rings assembled with the slider. The moving mass of the finger ring is modeled as m_l. The effective moving mass of the actuator is modeled as m_m, and b_m is the viscous damping of the actuator. And k_{tr} is the transmission stiffness between the actuator and the slider. The transmission friction is ignored. The haptic device can be modeled as in Figure 3. The force and motion are f_h and $\dot{x}_h$ respectively at the human operator driving point□and f_m and $\dot{x}_m$ at the actuator driving point.

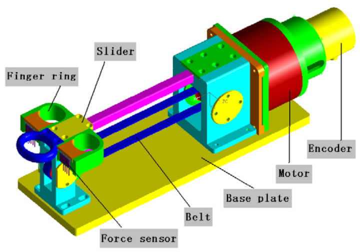

Fig. 2. A 1-DOF haptic device

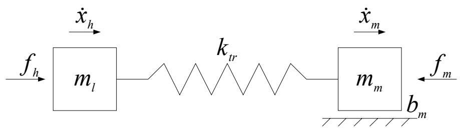

Fig. 3. The haptic device model

The dynamic equations of the system are

$$\begin{cases} f_h - k_{tr}\left(x_h - x_m\right) = m_l \ddot{x}_h \\ k_{tr}\left(x_h - x_m\right) - f_m = m_m \ddot{x}_m + b_m \dot{x}_m \end{cases} \quad \textbf{(1)}$$

The Laplace transformation of the equations is

$$\begin{cases} f_h(s) - \frac{k_{tr}}{s}\left(\dot{x}_h(s) - \dot{x}_m(s)\right) = m_l s \dot{x}_h(s) \\ \frac{k_{tr}}{s}\left(\dot{x}_h(s) - \dot{x}_m(s)\right) - f_m(s) = m_m s \dot{x}_m(s) + b_m \dot{x}_m(s) \end{cases} \quad \textbf{(2)}$$

where s is the Laplace operator.

This paper will follow the analogy between the mechanical and electrical system [8,11,12], which relates force to voltage and velocity to current. The homologous relationship of elements is described as: mass to inductance, damping to resistance, and spring to capacitance. The system analogy must obey the rule that the dynamic equations of two analogous systems must have the same form. According to [13], the human impedance can be considered to be passive, and in this paper the virtual environment is also assumed to be passive.

Using the device shown in Figure 2 as a haptic interface to implement an admittance display, an analogous electrical network system is structured as Figure 4. In figure 4, the left side of the dashed line 1 is the model of the human operator. It is a parallel combination of an active part and a passive impedance. The active part of the human operator $\dot{X}_h$ acts as the desired input of the system, and is analogous to be an independent flow source, which is in parallel with passive human impedance Z_h. And $\dot{x}_h$ is the actual motion at the human operator driving point. The elements between the dashed line 1 and 2 are the electrical network of the haptic device shown in Figure 3. The right side of the dashed line 3 is the virtual environment. The virtual environment is assumed to be passive with impedance $Z_e = M_e s$, where M_e is the mass of the virtual environment.

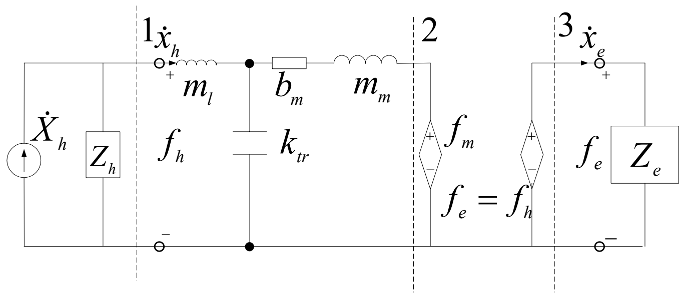

Fig. 4. The analogous electrical system

The measured force at the human operator driving point is the input of the virtual environment

$$f_e = f_h \tag{3}$$

The response of the virtual environment is $\dot{x}_e$. A simple proportional velocity control is considered. The control law is defined as

$$f_m = ZOHK_v(\dot{x}_h - \dot{x}_e) \tag{4}$$

where K_v is the proportional gain, and ZOH is the transfer function of the zero-order hold.

Combining equation (3), (4) and (2), the force and motion relation between the human operator and the virtual environment is

$$\left[m_l s+\frac{k_{tr}(m_m s+b_m)}{m_m s^2+b_m s+k_{tr}}+\frac{k_{tr}K_v}{m_m s^2+b_m s+k_{tr}}ZOH\right]\dot{x}_h=f_h+\frac{k_{tr}K_v ZOH}{m_m s^2+b_m s+k_{tr}}\dot{x}_e \tag{5}$$

and

$$ZOH=\frac{1-e^{-Ts}}{s}\approx\frac{2}{2+sT} \tag{6}$$

where T is the sampling rate.
The admittance at the human operator driving point is

$$Y_{eq}=\frac{\dot{x}_h}{f_h}=\frac{1}{A}+\frac{B}{AZ_e} \tag{7}$$

where

$$A=m_l s+\frac{k_{tr}(m_m s+b_m)}{m_m s^2+b_m s+k_{tr}}+\frac{k_{tr}K_v}{m_m s^2+b_m s+k_{tr}}ZOH \tag{8}$$

and

$$B=\frac{k_{tr}K_v ZOH}{m_m s^2+b_m s+k_{tr}} \tag{9}$$

The resulting transfer function of the system is

$$\frac{\dot{x}_h}{\dot{X}_h}=\frac{Z_h Y_{eq}}{1+Z_h Y_{eq}} \tag{10}$$

3 Stable criteria and analysis

Two-port model is used for the analysis of circuits in which bi-directional energy flows are present at two pairs of terminals. In figure 4, the part between the dashed lines 1 and 3 can be considered as a two-port system. Therefore, two-port theory can be applied to analyze the stability of the haptic system [8,11].

3.1 Stable criteria

For an arbitrary two-port, assuming the immittance matrix to be

$$\begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} \tag{11}$$

where possible matrices can be the impedance matrix, the admittance matrix, and the hybrid matrix. The two-port is absolutely stable if and only if the matrix satisfies

$$\begin{cases} \mathrm{Re}(p_{11}) \geq 0 \ \&\ \mathrm{Re}(p_{22}) \geq 0 \\ 2\,\mathrm{Re}(p_{11})\mathrm{Re}(p_{22}) \geq |p_{12}p_{21}| + R_e(p_{12}p_{21}) \forall \omega \geq 0 \end{cases} \tag{12}$$

where $\mathrm{Re}(p_{ij})$ is the real part of p_{ij}. Absolutely stable means that the two-port must be stable for any set of passive one-port [6]. In the application of a haptic system, the theory of absolute stability promises an effective method to solve the parameter range of virtual environment, within which the system can be stable under any passive human impedance.

The hybrid matrix of the haptic system shown in figure 4 is

$$\begin{bmatrix} \dot{x}_h \\ f_e \end{bmatrix} = \begin{bmatrix} Y_{eq} & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} f_h \\ -\dot{x}_e \end{bmatrix} \tag{13}$$

Applying equation（12），the criteria for absolute stability is

$$\mathrm{Re}(Y_{eq}) \geq 0 \tag{14}$$

Y_{eq} includes the parameters of both the haptic device and the passive virtual environment. From it the sufficient and necessary condition for absolute stability under arbitrary passive human impedance can be derived.

3.2 Stable boundary of virtual mass

We define a stable boundary as a limit of virtual environment parameter within which the system will be stable. When the virtual environment is considered to be a pure mass, the stable boundary of virtual mass is used to evaluate the performance of the system under arbitrary passive human impedance. For the system shown in figure 4, we analyze this mass boundary and factors affecting it.

Applying equation (10) to equation□14□□the mass range is

$$M_e \geq 2K_v \frac{k_{tr} b_m (CT + 2b_m) + \left[m_l \left(C^2 + b_m{}^2 \omega^2\right) + k_{tr}\left(Cm_m - b_m{}^2\right)\right]\left(2C - b_m T\omega^2\right)}{\left(C^2 + b_m{}^2\omega^2\right)\left[k_{tr} b_m \left(4 + T^2\omega^2\right) + 2K_v \left(2C - b_m T\omega^2\right)\right]} \quad \forall \omega \in [0, \infty) \tag{15}$$

where

$$C = k_{tr} - m_m \omega^2 \tag{16}$$

When $\omega = 0$ □equation□15□can be further reduced to

$$M_e \geq K_v \frac{2(m_m + m_l) + b_m T}{2(b_m + K_v)} \tag{17}$$

We can speculate factors affecting the mass range from equation □17□□

1. The mass range is independent of the transmission stiffness;
2. Reducing the moving mass of haptic device can augment the stable ability of the system;
3. Reducing T can enlarge the mass range;
4. Reducing the proportional gain will enlarge the mass range;
5. Observing the derivative of the right side of expression (17) with respect to b_m, it shows that enhancing the damping will enlarge the mass range when $TK_v - 2(m_m + m_l) < 0$.

4 Simulation

To prove the proposed method and the results, simulation was carried out using the control system described in equation (10). The control system is modeled as in Figure 5. In this model, Z_h is the human operator impedance, and Y_{eq} is the admittance at the human operator driving point described in equation (7). In the simulation, a step signal $\dot{X}_h$ acts as the system input.

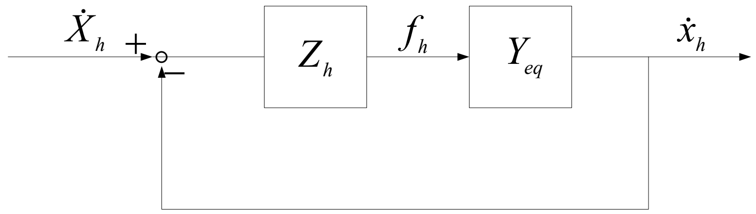

Fig. 5. Model of the control system

Assuming $b_m = 1N\sec/m$, $K_v = 0.2N\sec/m$, $m_m = 0.4kg$, $m_l = 0.5kg$, $T = 1m\sec$, $k_{tr} = 80\times10^3 N/m$, the mass range is $M_e \geq 0.15kg$ according to equation (17), which means that when $M_e \geq 0.15kg$ the system is stable for arbitrary human parameters, and when $M_e < 0.15kg$ the system may not be stable for a large spectrum of human impedance. Assuming an arbitrary human impedance $Z_h = \frac{50N/m}{s}$, figure 6 and Figure 7 is the step response of the system when $M_e = 0.14kg$ and $M_e = 0.15kg$ respectively. Figure 6 shows the step response divergent, and Figure 7 shows the step response critical stable. The simulation proves the model and the analytical result.

Fixing other parameters, altering the system parameters k_{tr} □ m_m and b_m etc, the effect of those parameters to the stable boundary meet well with what equation (17) demonstrates. For example, when $m_m = 0.3kg$ and $K_v = 0.1N\sec/m$, the step responses at the point $M_e = 0.14kg$ are all convergent as in figure 8 and figure 9, which demonstrate that reducing the moving mass, decreasing the proportional gain will enlarge the mass range of the virtual environment.

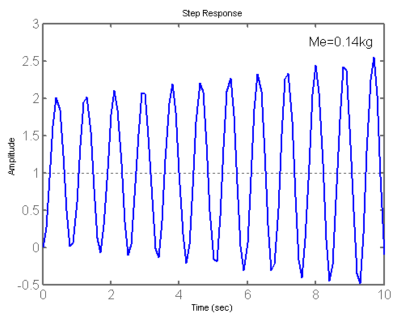

Fig. 6. system unstable when $M_e < 0.15kg$

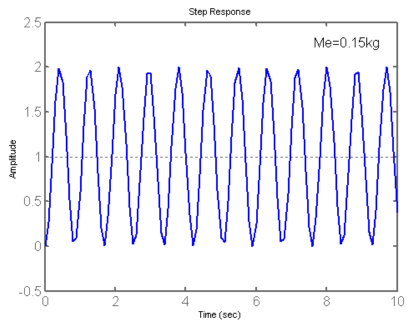

Fig.7. system stable when $M_e \geq 0.15kg$

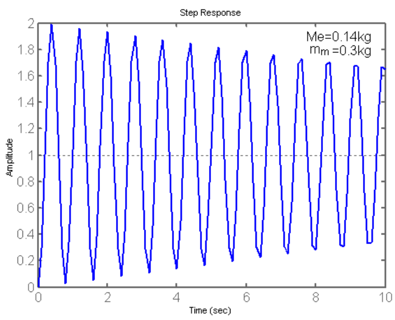

Fig. 8. stable boundary enlarged when reducing the moving mass

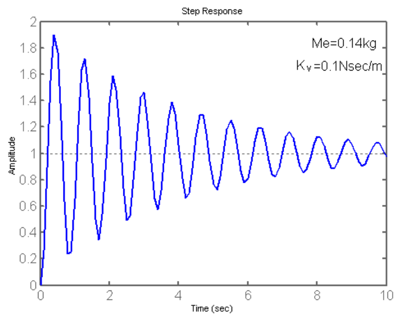

Fig. 9. stable boundary enlarged when reducing the proportional gain

5 Conclusion

This paper provides a method to the stability analysis for a haptic admittance display system, when interacting with a passive virtual environment.

For pure mass virtual environment, this paper provides the mass range within which the system can be stable under any passive human impedance. Factors affecting that range are analyzed. The stable boundary is independent of the transmission stiffness. Reducing the moving mass of the haptic device and the proportional gain of the controller will both enlarge the mass range. Enhancing the sampling rate and the system damping will also enlarge the mass range. The results of this paper provide a guidance for the design of 1-DOF haptic devices, and form the foundation of multi-DOF haptic system design and analysis.

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References

1. Thomas H. Massie and J. K. Salisbury, "The PHANTOM Haptic Interface: A Device for Probing Virtual Objects," Proc. Of the ASME Winter Annual Meeting, Symposium on Haptic Interfaces for Virtual Environment and Teleoperator Systems, Chicago, IL, Nov. 1994
2. J. Edward Colgate and J. Michael Brown, "Factors Affecting the Z-Width of a Haptic Display," Proc. Of the IEEE Int. Conf. On Robotics and Automation, pp. 3205-3210, 1994
3. Greg R. Luecke and Yong-Ho Chai, "Stability and Performance Comparison of the Force Reflecting Haptic Manipulator," Proc. Of the IEEE Int. Conf. On Robotics and Automation, pp. 637-642, 1997
4. Van der Linde R. Q., Lammertse P., Frederisksen E., Ruiter B., "The HapticMaster, a new high-performance haptic interface," Proc. Eurohaptics 2002, p. p. 1-5, July 8-10, Edinburgh, UK
5. Clover C. L., Luecke G. R., Troy J. J. and McNeely W. A., "Dynamic Simulation of Virtual Mechanisms with haptic feedback using industrial robotics equipment," Proc. Of the IEEE Int. Conf. On Robotics and Automation, pp. 724-730, 1997
6. Yokokohji Y., Hollis R. L. and Kanade T., "Vision-based visual/haptic registration for WYSIWYF display," Proceedings of the IEEE/RSJ Int. Conf. On Intelligent Robots and Systems, pp. 1386-1393, 1996
7. J. Edward Colgate and Gerd Schenkel, "Passivity of a Class of Sampled-Data Systems: Application to Haptic Interfaces," Proc. Of the American Control Conference, pp. 3236-3240, June 1994
8. Richard J. Adams and Blake Hannaford, "Stable Haptic Interaction with Virtual Environments," IEEE Transactions on Robotics and Automation, Vol. 15, No. 3, pp. 465-474, June 1999

9. R. W. Daniel and P. R. McAree, “Fundamental Limits of Performance for Force Reflecting Teleoperation,” the International Journal of Robotics Research, Vol. 17, No. 8, pp. 811-830, August 1998
10. Luo Yangyu, Wang Dangxiao and Zhang Yuru, “Development of a Haptic Display Interface,” 2nd International Symposium on Instrumentation Science and Technology, pp. 272-276, 2002
11. Blake Hannaford, “A Design Framework for Teleoperators with Kinesthetic Feedback,” IEEE Transactions on Robotics and Automation, Vol. 5, No. 4, pp. 426-434, August 1989
12. Robert J. Anderson, “Building a Modular Robot Control System using Passivity and Scattering Theory,” Proc. Of the IEEE Int. Conf. On Robotics and Automation, pp. 698-705, 1996
13. Neville Hogan, “Controlling Impedance at the Man/Machine Interface,” Proc. Of the IEEE Int. Conf. On Robotics and Automation, pp. 1626-1631, 1989