

Touching trajectories: the relation between speed and curvature in exploring shape.

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Abstract

Studies in motor psychophysics have documented that the tangential velocity of movement decreases with increasing curvature of the path being followed. Views on the origin of this relation range from higher order motion planning processes such as trajectory selection according to a principle of minimisation of mean squared rate of change of acceleration (jerk) to lower level motor constraints, for instance sinusoidal constraint on joint motion. We have examined finger-scanning movements in which eight subjects traced repeatedly round various raised outline shapes. We were interested to see whether velocity would be related to curvature from the outset, suggesting low level sensorimotor constraints, or whether the relation would emerge with experience, as a result of increased trajectory planning. We found evidence of a relation between speed and curvature from the outset, in support of lower level, rather than trajectory planning, constraints.

1. Introduction.

A fundamental law in motor psychophysics is the one third power relation between movement speed (or more precisely, tangential velocity) and radius of curvature of the path followed:

$$\text{speed} = \text{radius_of_curvature} * \exp(1/3).$$

The smaller the radius of curvature (the greater the curvature) the lower the speed.

Although a relation between speed and curvature has been acknowledged for over a century (Jack, 1895) there has been considerable growth in interest since Lacquaniti et al (1983) provided a thorough quantification of the law in drawing movements. The law holds for writing as well as drawing movements. It holds for movements in the plane or in 3d (Schall and Sternad, 2001) and for drawing motions directed by 3-d isometric force control (Massey et al, 1992). The one-third power law has been demonstrated in children as well as adults although the exponent changes with age

(Viviani and Schneider, 1991). It has been found for mechanically constrained or free form movements, with or without visible trace of the path traced out (Viviani, BaudBovy & Redolfi, 1997) and for eye movements (de Sperati and Viviani, 1997). When participants were guided around an elliptical track by a robot, which followed a different relation between speed and curvature, subsequent free reproduction of the track without any guide conformed to the one third power law (Viviani et al 1997).

The one-third power law has been attributed to motion planning processes with trajectory selected according to a principle of minimisation of jerk, the third derivative of position, in order to obtain maximally smooth movements (Flash and Hogan, 1985; see also Wann, Nimmo-Smith & Wing, 1988; Viviani and Flash 1995). An alternative interpretation is that the relation reflects a fundamental constraint on motor control. For example, in handwriting it has been attributed to the combination of oscillatory tendencies related to x- and y-directions which may reflect joint rather than hand space constraints (Schall and Sternad, 2001).

In the above studies the movements analysed were made to spatial targets which the participant had opportunity to plan. We ask whether the one-third power law applies to tactile exploration where a continuous curve target path (circle and ellipses) has to be discovered. When finger scanning movements are used to trace an outline shape without visual guidance, it is reasonable to expect that the demands of tactile and proprioceptive information pick-up will result in relatively slow movements. But, will the slowing be related to curvature? If the one-third power law arises from planning processes, the relation between speed and curvature would only be expected to appear as information about the path becomes available from the haptic input. However, if the relation arises from low level sensorimotor constraints, these would be expected to scale down to lower speeds used in first exploring the shape. The results that we report in this paper show differences in speed at the major and minor axes of the ellipses

consistent with the power law thus supporting sensorimotor rather than motion planning constraints.

2. Method.

2.1 Subjects.

Eight subjects (4 male, 4 female, age range 19 to 22 years, right-handed for writing), students of The University of Birmingham, participated on a voluntary basis.

2.2 Apparatus.

Motion of a reflective marker placed on the nail bed of the index finger of the right hand was tracked at 100 Hz with a 3-camera infrared motion tracking system (Qualisys ProReflex). The system calibration yielded 0.2 mm resolution over a working area of 40 x 40 cm. Outline shapes were constructed from (4 mm diameter) plastic coated electrical cable glued to a stiff backing board. They comprised a circle (radius 12 cm), a wide ellipse (angular eccentricity 45° and linear eccentricity 24 cm) and a narrow ellipse (25° and 24 cm).

2.3 Task and procedure.

The subject was seated at a table with the outline shape placed on it so that the semiaxes (of the ellipse) were aligned to the subject's coronal (left-right) and sagittal (anterior-posterior) planes and corresponded to the x and y axes of the motion tracking system. Instructions were given to use the right index finger to trace each shape at a comfortable speed for 30 s in an anti-clockwise direction with the pad of the index finger on top of the wire. The subject was instructed not to rest the finger tip on the surface, nor to use the wire as a guide "fence", nor was any other part of the hand or arm allowed to touch the shape or surface of the table. Three successive trials were run with each shape before proceeding to the next shape. The ellipses were presented, first with long, then with short, axis parallel to the coronal plane. All trials were first run with the subject's eyes closed and subsequently repeated, one trial per condition, with the subject allowed full vision.

2.4 Data processing.

Tangential velocity (speed) at x- and y-extremes was determined from the finger tip trajectory after low-pass filtering (Butterworth 2-order 10 Hz) the x and y coordinate streams.

2.5 Design.

A repeated measures design was used with fixed order of outline shapes but with the order varied across subjects so that each shape appeared in first position approximately the same number of times.

3. Results.

Illustrative time and spatial finger tip trajectories for one subject tracing the wide ellipse oriented in the coronal plane without (above) and with (below) vision are shown in Figure 1. The time waveforms show the greater speed achieved in the presence of vision. The increased speed is associated with the development of a series of local speed maxima and these can be seen to be aligned with extremes in y-displacement associated with low curvature at each end of the minor axis of the ellipse. Emergence of similar maxima may be seen in the absence of vision, although, in this case, only after a couple of passes around the ellipse. In this example, the speed maxima are much more poorly defined than in the vision condition although there is a suggestion they align with the y extremes.

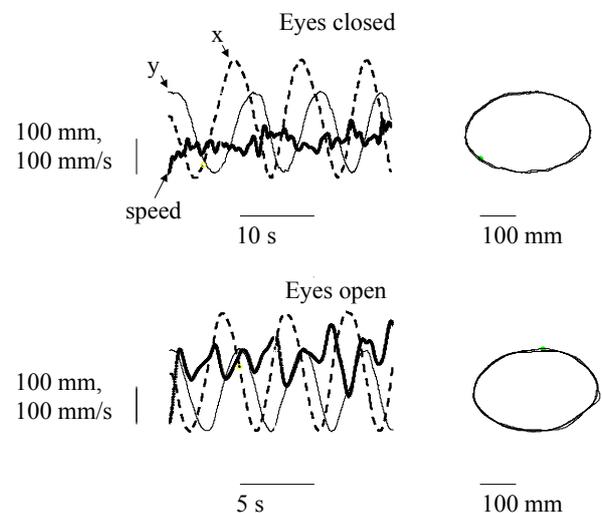


Figure 1: Time (left) and spatial(right) trajectories for two single trials in which the subject traced the wide ellipse by touch alone (above) or with vision (below). Displacements (x dashed, y thin continuous lines), speed (thick line) are plotted on the same vertical scale but note the difference in the time scale reflecting the doubling in speed with vision.

The number of cycles completed in each 30 s trial shown in Figure 2 provides an indication

of the average speed of finger scanning. It can be seen that, for each outline shape and ellipse orientation, the speed progressively increases across the 3 trials without vision, although remaining slower than with vision. This was confirmed by reliable main effects of trial ($F_{3,21}=53.49, p<.01$) and stimulus ($F_{4,28}=5.65, p<.01$) in repeated measures ANOVA. The interaction between trial and stimulus was also reliable ($F_{12,84}=4.92, p<.01$).

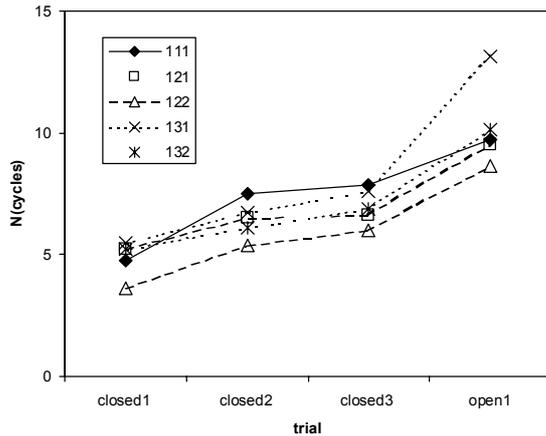


Figure 2. Average ($N=8$ subjects) of the number of cycles completed in each 30 s trial as a function of trial (trials 1-3 eyes closed, 4 eyes open). Five stimulus types are represented in the five sets of lines - circle (coded 111), wide (12.) or narrow (13.) ellipse with the ellipses oriented with major axis parallel to the coronal (.1) or saggital (.2) plane.

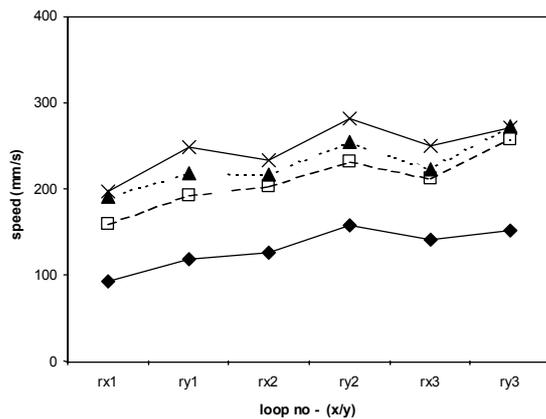


Figure 3. Speed as a function of 3 successive loops (measures taken at x and y extremes in alternation) around the circle. The four curves relate to no vision trials 1-3 (lower speeds) and vision trial 4 (highest speed).

The measured speed at x and y displacement extremes in tracing around the circle for the first three complete loops is

shown in Figure 3. An increase in speed over haptic trials is clear although the trials with vision remain faster. There is also an increase in speed evident over the 3 successive loops around the circle.

The speed at major and minor axes of the wide and narrow ellipses is shown in Figure 4. An increase in speed over haptic no vision trials is clear although the trials with vision remain faster. There is also a slight increase in speed evident over the 3 successive loops. It can also be seen that the speed is greater for the lower curvature minor axis of the ellipse which, depending on whether the ellipse is oriented with coronal (LR) or saggital (AP) planes, corresponds to the y (in LR) or x (in AP) displacement extremes.

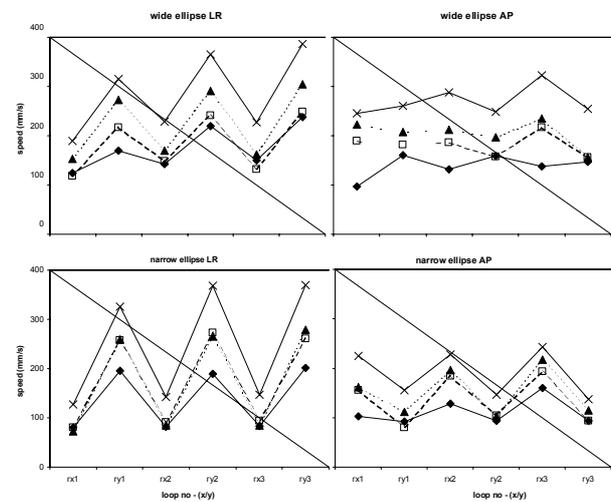


Figure 4. Speed as a function of 3 successive loops (measures taken at x and y extremes in alternation) around the ellipse. In each of the four plots the curves relate to no vision trials 1-3 (lower speeds) and vision trial 4 (highest speed). The separate plots relate to ellipse shape (top row - wide; bottom row - narrow) and orientation: (left column coronal LR; right column saggital (AP)).

Separate repeated measures ANOVAs for each stimulus generally revealed significant main effects (9 out of 12 cases) of trials, loops and axis ($F_{3,21}, F_{2,21}$ and $F_{1,7}$, ranging 8.57-38.88, $p<.01$, 3.89-16.86, $p<.05$ and 32.80-135.03, $p<.01$ respectively). Interaction terms involving loop number x axis and trial x axis ($F_{2,14}$ and $F_{3,21}$ ranges 6.34-32.51, $p<.01$ and 5.71-17.82, $p<.01$ respectively) were significant in 7 out of 8 cases due to there being less difference between speed at high and low curvatures at x,y extremes in the initial trial and initial loop. However, none of the three-way trial x loop x axis interactions was significant.

4. Conclusion.

Numerous studies have shown a power law relation exists between curvature and tangential velocity (speed). For the first time we have examined haptic exploratory movements in which subjects used the index finger to trace out a raised outline. We considered two alternative predictions. Under the motion planning hypothesis we predicted that in the early stages speed would be constant. Only with knowledge of shape gained from haptic exploration would the relation between speed and curvature emerge. Under motor constraints we expected that the relation between speed and curvature would hold from the outset, even at the lower speeds of initial exploration.

The results showed large increases in the speed of scanning with the index finger tip with growing experience of moving around circular and elliptical outline shapes. However, a relation between speed and curvature was observed from the very beginning, although the difference in speed at high and low curvatures was somewhat less in initial loop around the ellipses and on the initial trial. We therefore suggest that the one third power law reflects low level sensorimotor processes involved in movement execution and not higher level motion path planning processes. However, this conclusion must be tempered by the following observations:

- 1) in this preliminary analysis we have not documented the continuous relation between speed and curvature, but simply focussed on curvature extremes.
- 2) in this study, the regularity of the geometric shapes (circles and ellipses) may have led to rapid recognition of the particular form being traced. This means that subjects may have been able to form a mental image of the general shape class, sufficient to support motion planning very soon after contact. In further experiments we are evaluating haptic scanning of irregular raised outline paths of non-closed form.

5. References.

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